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EVAPORATION OF THE MICROLAYER IN HEMISPHERICAL BUBBLE GROWTH IN NUCLEATE BOILING OF LIQUID METALS*†

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Abstract—In this paper, a thermal analysis is used to estimate the extent of evaporation of the microlayer in hemispherical bubble growth, in nucleate boiling of liquid metals on heated surfaces. As the bubble grows, evaporation of the microlayer produces a dry patch at its center, whose size depends on the thermal and physical properties of the system, the roughness of the heating surface, and the boiling pressure. It was found that the area of this patch relative to that of the microlayer (or bubble base) is typically very small for liquid metals, and can be neglected in most theoretical analyses of bubble growth. It was further found that the loss of liquid from the microlayer due to evaporation into the bubble is at most a few percent, in a typical case.

Since both the calculational model and mathematical analysis involve a number of simplifying assumptions, the numerical results of this pioneering study should be considered approximate.

NOMENCLATURE

A_{cs}, $2\pi R^2$ = area of the hemispherical surface of a hemispherical bubble [ft^2];
C_0, coefficient in equation (2) [ft/h^n];
C_{pL}, specific heat of liquid [Btu/(lb$_m$. °F)];
C_{pw}, specific heat of heating solid (or wall) [Btu/(lb$_m$. °F)];
H, $K\sqrt{\nu}$ = coefficient in equation (4) [ft/h$^{\frac{1}{2}}$];
K, coefficient in equation (1) [dimensionless];
k_L, thermal conductivity of liquid [Btu/(h . ft . °F)];
k_w, thermal conductivity of heating solid (or wall) [Btu/(h . ft . °F)];
n, exponent in equation (2) [dimensionless];
p_∞, pressure in liquid remote from the growing bubble [atm];
Q_{bb}, total heat rate to hemispherical bubble at time θ [Btu/h];
Q_{ml}, heat rate from microlayer to hemispherical bubble at time θ [Btu/h];
Q_{cs}, heat rate from curved surface to hemispherical bubble at time θ [Btu/h];
r, radial distance (see Fig. 2) [ft];
R, bubble radius at time θ [ft];
r_e, radius of dry-patch (see Fig. 2) [ft];
r_0, bubble radius at start of growth ($\theta = 0$) [ft];
S_n, quantity defined by equation (20) [dimensionless];
$S_{\frac{1}{2}}$, value of S_n when $n = \frac{1}{2}$ [dimensionless];
S_1, value of S_n when $n = 1$ [dimensionless];
$S_{0\cdot9}$, value of S_n when $n = 0{\cdot}9$ [dimensionless];
t_{sat}, saturation temperature corresponding to p_∞ [°F];
t_v, temperature of vapor in bubble at time θ [°F];
t_{wo}, temperature of heating surface at start of bubble growth ($\theta = 0$) [°F];
V_L, total volume of liquid evaporated from the microlayer up until the bubble has grown to radius R [cm^3 or ft^3];
V_{mo}, total volume of microlayer formed during the time the bubble has grown to radius R [cm^3 or ft^3];
V_v, volume of vapor produced from V_L [cm^3 or ft^3].

Greek symbols

α_L, thermal diffusivity of liquid [ft^2/h];
α_w, thermal diffusivity of heating solid (or wall) [ft^2/h];
δ, microlayer thickness at time θ and radial distance r [ft];
δ_0, initial thickness of microlayer at time θ_g and radial distance r [ft];
θ, time after start of bubble growth [h];
θ_g, time required for bubble to grow to radius r less than R [h];
λ, latent heat of vaporization [Btu/lb$_m$];
μ, molecular viscosity of liquid [lb$_m$/(h . ft)];
ν, kinematic viscosity of liquid [ft^2/h];
ξ_n, quantity defined by equation (17) [ft/(h$^{\frac{1}{2}}$. °F)];
$\xi_{\frac{1}{2}}$, value of ξ_n when $n = \frac{1}{2}$ [ft/(h$^{\frac{1}{2}}$. °F)];
ξ_1, value of ξ_n when $n = 1$ [ft/(h$^{\frac{1}{2}}$. °F)];
$\xi_{0\cdot9}$, value of ξ_n when $n = 0{\cdot}9$ [ft/(h$^{\frac{1}{2}}$. °F)];
ρ_L, density of liquid [lb$_m$/ft^3];
ρ_v, density of vapor [lb$_m$/ft^3];
ρ_w, density of heating solid (or wall) [lb$_m$/ft^3].

INTRODUCTION

IN HEMISPHERICAL bubble growth in nucleate boiling on a heated surface, a very thin layer of liquid remains on the heated surface under the bubble. The thickness of this microlayer approaches zero at the point of

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bubble nucleation, but as the bubble grows the microlayer formed at its periphery gradually increases in thickness. The hydrodynamics of its formation has been studied by Cooper and Lloyd [1], Olander and Watts [2], and Dwyer and Hsu [3]. As a hemispherical bubble grows, heat transfer from the heating surface causes the microlayer to evaporate into the bubble, thereby reducing its thickness. The result of this is that in the central region of the microlayer a dry-patch is formed, the final size of which depends on the characteristics of the boiling system.

Because of the inapplicability of optical techniques to liquid metals, direct measurement of dry-patch areas with these liquids have not been made. We therefore must resort to theoretical estimates, and the present paper presents the results of the first known study with this objective.

Actually, it appears that very few dry-patch measurements have yet been made with ordinary liquids, where the results are sufficiently accurate and extensive to present a clear picture. Although the results of such studies are not directly applicable to liquid metals, they do have some relevance, and a few cases may be briefly mentioned.

Sharp [4] was the first experimenter to observe the appearance of a dry-patch in the center of the microlayer, when he boiled water on a polished flint-glass surface that contained a tiny scratch to initiate nucleation. In an experiment designed to allow the vapor bubbles to grow normally (i.e. without deformation), he observed that when the time-averaged heat flux was great enough, which he estimated to be in the range 10000 to 15000 Btu/(h.ft^2), dry spots appeared after about 5–10 ms, grew with time, and then disappeared at bubble release.

Jawurek [5] boiled methanol on a transparent film of stannic oxide on glass and measured dry-patches by using parallel monochromatic illumination from below. He recorded interference patterns by high-speed photography, which gave a direct measure of dry-patch size. Unfortunately, he reported quantitative results for only a single run, for which the pressure was 180 in Hg abs., the time-average heat flux was 19 700 Btu/(h.ft^2), and the subcooling was 12·8°F. The results showed that the dry-patch started to grow at the instant of bubble inception and continued to grow to a radius of ~ 2·8 mm 33·4 ms later, when the radius of the bubble had reached ~ 7·5 mm.

More extensive measurements of dry-patch size and growth rate were made by Cooper and Lloyd [6] by boiling toluene on glass at pressures of 1 and 2 lb/in$^2_{\text{abs}}$. They found that the dry-patch area relative to that of the microlayer (base area of bubble) was quite small, varying from ~ ½ per cent at the beginning of bubble growth to ~ 4 per cent at the end of the hemispherical-bubble-growth period. These results were obtained under slightly subcooled conditions (2–3°F), but they observed that subcoolings up to 14°F had no significant effects on the size and growth rate of the dry-patch area, even though the bubble size was considerably reduced.

ANALYSIS

Any tractable theoretical analysis of the growth of the dry-patch must necessarily involve a number of simplifying assumptions, so that the results at best can only be considered approximate. Cooper and Lloyd [1] developed an analytical method for estimating the dry-patch size in nucleate boiling of ordinary liquids, but it is inapplicable to liquid metals mainly for two reasons: first, they assumed the temperature of the vapor inside the bubble remained constant at t_{sat}, which implies heat-transfer-controlled growth; and second, they assumed that the heating-surface temperature remained constant during bubble growth, with a linear temperature gradient through the microlayer.

The following analysis gives an estimate of the dry-patch radius r_e at anytime θ, as a function of the radius R of a hemispherical bubble, in nucleate boiling of a liquid metal on a flat horizontal surface.

Dwyer and Hsu [3] have shown that microlayer thicknesses for liquid metals are much less than those for ordinary liquids. They showed that a good approximation of the microlayer thickness at time of formation is given by the relation

$$\delta_0 = K(\nu\theta_g)^{\frac{1}{2}}, \tag{1}$$

where K is a function of n in the standard empirical equation of bubble growth

$$R = C_0\theta^n. \tag{2}$$

The coefficient C_0 is a function of n, and n varies between the limits of $\frac{1}{2}$ (for heat-transfer-controlled bubble growth) and 1 (for inertia-controlled growth). Bubble growth rates are generally much faster for liquid metals (where n is usually nearer 1 than $\frac{1}{2}$) than for ordinary liquids (where n is usually nearer $\frac{1}{2}$ than 1). For example, it is estimated [7] that the bubble growth time for sodium boiling at 1 atm is of the order of 1 ms, when the bubble nucleation radius r_0 is 5×10^{-4} in, and when the heating surface is the top side of a stainless-steel plate. Under these conditions, n equals 0·95 and $K = 0{\cdot}62$ [3], and according to equation (1) the maximum thickness of the microlayer is about 0·01 mm. The diffusion time for this thickness will be of the order of (thickness)$^2/\alpha_L$, or only $\sim 2 \times 10^{-3}$ ms.

We shall therefore assume that the thickness of the microlayer is negligibly small compared with that of the heater plate, that its heat capacity is negligibly small compared with the rate of heat transfer through it, and that the temperature at any point on the solid–liquid interface may be taken equal to the temperature t_v of the vapor in the bubble the moment the bubble grows over that point.

We further assume that the thermal conductivity of the liquid is at least moderately high compared with that of the heater plate. These conditions, believed to approximate those for many boiling liquid-metal systems, call for negligible temperature drop through the microlayer compared with that in the heater plate near the heating surface. They also indicate negligible thermal time lag caused by the microlayer, as the bubble expands over a given point on the heating surface.

We next make three additional assumptions that are commonly made in bubble-growth analyses. The first of these is that the vapor in the bubble is in thermodynamic equilibrium with its bounding liquid surface; the second is that the heating surface is perfectly wetted; and the third is that the variation of the vapor temperature t_v during most of the bubble's growth is not large compared to $(t_{wo}-t_v)$, where t_{wo} is the temperature of the heating surface at $\theta = 0$. Regarding this last assumption, the pressure (and therefore t_v) drops very rapidly during the very early stage of bubble growth due to work of expansion against acceleration and surface-tension forces, which soon become negligible. Thereafter, the pressure drops much more slowly, because the work of expansion is consumed only in pushing back the liquid.

The thickness of the outer layer of the heating plate in which the temperature fluctuates during boiling is known as the penetration length. It is assumed that this distance is sufficiently short [4] and the temperature gradient therein sufficiently flat [8] at $\theta = 0$ (i.e. at start of bubble growth) that the temperature over the distance L can be considered uniform at t_{wo} at $\theta = 0$. It is further assumed that the temperature of the liquid in which the hemispherical bubble starts to grow is also uniform at t_{wo}. There are two arguments in support of this assumption. First, because of the long "waiting" periods (i.e. the time between the departure of one bubble and the initiation of the next at the same nucleation site) with, and high thermal conductivities of, liquid metals, the typical thermal boundary layer is relatively thick and the temperature gradients therein relatively flat. This can be shown from the analysis of Mikic *et al.* [9] for growth of spherical bubbles on heated solid surfaces. Using their generalized equation for bubble growth, it is found [7] that, in the case of sodium boiling at 1 atm and with an initial wall superheat $(t_{wo}-t_{sat})$ of 100°F, a waiting time of 1 s gives bubble growth rates that are very nearly as high as those obtained with an infinite waiting time (i.e. flat temperature profile in the liquid at $\theta = 0$). Waiting times of the order of 1 s are rather typical for liquid metals [8, 10].

The second argument in support of the assumption that the temperature of the liquid in which the bubble starts to grow is uniform at t_{wo} is that the bubble pushes the liquid thermal boundary layer ahead of it as it grows (Fig. 1).

Finally, we assume unidirectional heat flow through the heating plate and through the microlayer to the bubble. Under these conditions, and on the basis of some of the assumptions mentioned above, the transient heat conduction from the solid (and therefore the heat flux through the microlayer) is similar to that where the flat face of a semi-infinite, uniform-temperature solid is suddenly brought into perfect thermal contact with an infinite heat sink at lower temperature. The plate acts like a semi-infinite solid because of the shortness of the transient, which causes the penetration length to be short relative to the thickness of a typical heating plate.

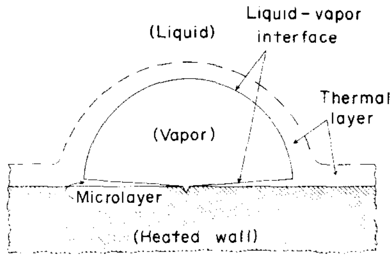

FIG. 1. Schematic drawing of hemispherical vapor bubble growing on a heated surface, in nucleate boiling of a liquid metal. For purpose of illustration, the relative thickness of the microlayer is much greater than in actuality.

From equation (2), we can write

$$r = C_0\theta_g^n, \tag{3}$$

where r is the bubble radius at anytime θ_g preceding θ, as illustrated in Fig. 2. And from equation (1), we can write

$$\delta_0 = H\sqrt{\theta_g}, \tag{4}$$

where $H \equiv K\sqrt{\nu} = a$ constant for a given boiling system.

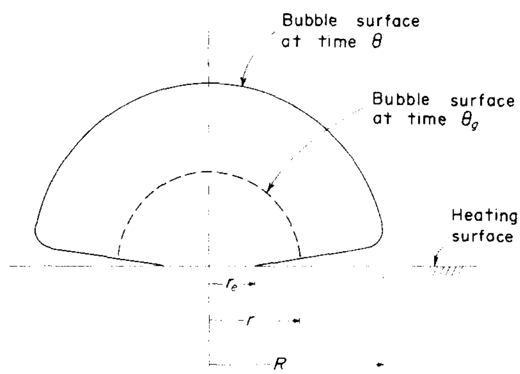

FIG. 2. Idealized calculational model of hemispherical bubble, showing microlayer and dry-patch on the heating surface at any time θ during growth. For purposes of illustration, the relative thickness of the microlayer and radius of the dry-patch are much greater than in actuality.

Prediction of dry-patch size

Because the heat flowing from the heating surface (and through the very thin microlayer) provides the latent heat for vaporizing the microlayer, we can write the heat-rate balance

$$-\rho_L\lambda\frac{d\delta}{d\theta} = \frac{k_w(t_{wo}-t_v)}{(\pi\alpha_w)^{\frac{1}{2}}(\theta-\theta_g)^{\frac{1}{2}}} \tag{5}$$

per unit area of microlayer, for any radial distance r, and for any time θ beyond θ_g. The RHS of equation (5) represents the local flux from the heater plate, which is analogous to the situation where a semi-infinite body at t_{wo} is suddenly brought into thermal contact with an infinite sink at t_v. We next proceed to eliminate $(t_{wo}-t_v)$ from equation (5) by getting an approximate expression for it in terms of R and θ. We do this by making a heat balance on the bubble at any time θ. This balance is

$$Q_{bb} = Q_{ml} + Q_{cs}, \tag{6}$$

where

Q_{bb} = rate of heat gain by the bubble,
Q_{ml} = heat rate from the microlayer to the bubble, and
Q_{cs} = heat rate from the curved surface to the bubble.

Since the rate of bubble growth depends on the rate of evaporation at its liquid boundaries, and since this in turn depends on the total rate of heat gain, we may, neglecting variations in ρ_v and λ, write the heat-rate equation

$$Q_{bb} = 2\pi\rho_v\lambda R^2 \frac{dR}{d\theta}, \tag{7}$$

which gives us an expression for Q_{bb} in terms of the variables R and θ.

Let us now turn our attention to Q_{ml}. From equation (5), we see that the local heat flux at radial distance r and time θ is given by

$$q_{ml} = \frac{k_w(t_{wo}-t_v)}{(\pi\alpha_w)^{\frac{1}{2}}(\theta-\theta_g)^{\frac{1}{2}}} \tag{8}$$

which we can re-write in the form

$$q_{ml} = \frac{(C_0)^{1/(2n)}k_w(t_{wo}-t_v)}{(\pi\alpha_w)^{\frac{1}{2}}(R^{1/n}-r^{1/n})^{\frac{1}{2}}}, \tag{9}$$

by substituting $(R/C_0)^{1/n}$ for θ and $(r/C_0)^{1/n}$ for θ_g. The instantaneous heat rate Q_{ml} from the heating surface under the bubble may be safely represented by

$$Q_{ml} = 2\pi\int_0^R q_{ml}\, r\, dr. \tag{10}$$

Substituting q_{ml} from equation (9) into (10) gives

$$Q_{ml} = \frac{2(C_0)^{1/(2n)}\pi^{\frac{1}{2}}k_w(t_{wo}-t_v)}{(\alpha_w)^{\frac{1}{2}}}\int_0^R \frac{r\, dr}{(R^{1/n}-r^{1/n})^{\frac{1}{2}}}, \tag{11}$$

the solution to which may be written in the form

$$Q_{ml} = \frac{2}{\theta^{\frac{1}{2}}}(\pi k_w C_{pw}\rho_w)^{\frac{1}{2}}(t_{wo}-t_v)R^2 n[B(2n, \tfrac{1}{2})], \tag{12}$$

where $B[(2n, \frac{1}{2})]$ represents a beta function in which the arguments are $2n$ and $\frac{1}{2}$. Values of $B(2n, \frac{1}{2})$ for different values of n over the whole range of n are given in Table 1. Notice that the product $n[B(2n, \frac{1}{2})]$, which appears in equation (12), only increases by 33 per cent as n increases from its lower limit of $\frac{1}{2}$ to its upper limit of 1.

Table 1. Values of the beta function $B(2n, \frac{1}{2})$ for various values of n

n	$B(2n, \frac{1}{2})$
0·5	2·000
0·6	1·791
0·7	1·635
0·8	1·513
0·9	1·413
1·0	1·333

A little thought will show that if the magnitude of the quantity $(k_L C_{pL}\rho_L)^{\frac{1}{2}}$ is close to that of $(k_w C_{pw}\rho_w)^{\frac{1}{2}}$, which it could well be for certain liquid-metal/solid-metal systems, the validity of equation (12) would not depend on the assumption that the heat capacity of the microlayer (because of its thinness) is negligible compared to the rate of heat transfer through it.

Let us now get an expression for Q_{cs} in terms of R, θ, and $(t_{wo}-t_v)$. We begin by writing the simple heat balance

$$Q_{cs} = q_{cs}A_{cs}, \tag{13}$$

where

q_{cs} = the heat flux at the hemispherically curved surface of the bubble, and
$A_{cs} = 2\pi R^2$ = the area of the hemispherical surface.

The equation for q_{cs} can be assumed to be the same as that for a spherical bubble growing in a uniformly heated liquid. From the asymptotic solutions of Plesset and Zwick [11] and Scriven [12], the heat flux for this situation is given by

$$q_{cs} = \frac{(\sqrt{3})k_L(t_{wo}-t_v)}{(\pi\alpha_L\theta)^{\frac{1}{2}}}, \tag{14}$$

which represents the transient heat case in which a semi-infinite body is suddenly exposed to an infinite spherically shaped heat sink at a lower temperature. After substituting equation (14) into equation (13), expressing A_{cs} in terms of R, and substituting $k_L/\rho_L C_{pL}$ for α_L, we finally get

$$Q_{cs} = \frac{2(3\pi)^{\frac{1}{2}}R^2(k_L\rho_L C_{pL})^{\frac{1}{2}}(t_{wo}-t_v)}{\theta^{\frac{1}{2}}}. \tag{15}$$

Now, substituting equations (7), (12), and (15) into (6), gives

$$t_{wo}-t_v = \frac{\theta^{\frac{1}{2}}}{\xi_n}\frac{dR}{d\theta}, \tag{16}$$

where

$$\xi_n = \frac{1}{\rho_v\lambda\pi^{\frac{1}{2}}}\{n[B(2n, \tfrac{1}{2})](k_w C_{pw}\rho_w)^{\frac{1}{2}} + (3k_L C_{pL}\rho_L)^{\frac{1}{2}}\} \tag{17}$$

and may be considered to be a parameter that characterizes the boiling behavior of the system.

We can now eliminate $(t_{wo}-t_v)$ from equation (5) by means of (16) and get

$$-d\delta = S_n\frac{\theta^{\frac{1}{2}}}{(\theta-\theta_g)^{\frac{1}{2}}}dR, \tag{18}$$

where

$$S_n \equiv \left(\frac{k_w C_{pw}\rho_w}{\pi}\right)^{\frac{1}{2}}\frac{1}{\xi_n\rho_L\lambda}. \tag{19}$$

Substituting equation (17) into (19) gives

$$S_n \equiv \frac{\rho_v/\rho_L}{n[B(2n, \frac{1}{2})] + \left[\dfrac{3k_L C_{pL}\rho_L}{k_w C_{pw}\rho_w}\right]^{\frac{1}{2}}}, \tag{20}$$

which is a more basic definition of the system parameter S_n.

If we now convert the independent variable of time in equation (18) to that of radius, by replacing $\theta^{\frac{1}{2}}$ by $(R/C_0)^{1/(2n)}$, θ by $(R/C_0)^{1/n}$, and θ_g by $(r/C_0)^{1/n}$, and then write the equation in integral form, we get

$$\delta_0 - \delta = S_n \int_r^R \frac{R^{1/(2n)}}{(R^{1/n} - r^{1/n})^{\frac{1}{2}}} \mathrm{d}R. \tag{21}$$

Combining equations (3) and (4) by eliminating θ_g shows that

$$\delta_0 = H\left(\frac{r}{C_0}\right)^{1/(2n)}; \tag{22}$$

and at the radius of the dry-patch, $r = r_e$, and $\delta = 0$. Making these substitutions in equation (21), gives

$$\frac{H}{S_n}\left(\frac{r_e}{C_0}\right)^{1/(2n)} = \int_{r_e}^R \frac{R^{1/(2n)}}{(R^{1/n} - r_e^{1/n})^{\frac{1}{2}}} \mathrm{d}R. \tag{23}$$

This equation cannot be readily integrated in terms of n; but it can be easily integrated, if n is taken either at its lower limit of $\frac{1}{2}$ or at its upper limit of 1. In liquid-metal boiling, the value of n is generally much nearer 1 than $\frac{1}{2}$, and at pressures below atmospheric n is practically 1 for essentially all of the bubble growth period [13]. For n to approach $\frac{1}{2}$ with liquid metals, they must be boiled at pressures well above atmospheric, something that is seldom done. Moreover, at supra-atmospheric pressures, the bubbles are small (at time of departure) and tend to be more spherical in shape [13]. However, if n were taken equal to $\frac{1}{2}$, it can be readily shown that the integration of equation (23) would lead to

$$\left.\frac{r_e}{R}\right|_{n=\frac{1}{2}} = \frac{1}{\left[1 + \left(\dfrac{H}{C_0 S_{\frac{1}{2}}}\right)^2\right]^{\frac{1}{2}}}. \tag{24}$$

which says that under these conditions r_e/R would be independent of R.

If n is taken at its upper limit of 1, equation (23) integrates to

$$\frac{H}{S_1}\left(\frac{r_e}{C_0}\right)^{\frac{1}{2}} = r_e\left\{\left(\frac{R}{r_e}\right)^{\frac{1}{2}}\left(\frac{R}{r_e} - 1\right)^{\frac{1}{2}} + \ln\left[\left(\frac{R}{r_e}\right)^{\frac{1}{2}} + \left(\frac{R}{r_e} - 1\right)^{\frac{1}{2}}\right]\right\}, \tag{25}$$

which gives the radius of the dry-patch (r_e) as a function of the bubble radius (R).

Let us now use equation (25) to calculate r_e/R as a function of R, for a case where n is clearly 1 throughout the entire bubble-growth period. Such a case [13] is sodium boiling under a pressure of 30 mm Hg on a smooth Type-316-stainless-steel plate, where $r_0 = 5 \times 10^{-4}$ in. For these conditions, it is estimated [13] that

$$\begin{aligned} t_{wo} - t_{sat} &= 239^\circ\mathrm{F}, \\ \xi_1 &= 12{\cdot}28\ \mathrm{ft/(h^{\frac{1}{2}}\,.\,^\circ F)}, \\ S_1 &= 1{\cdot}591 \times 10^{-5}, \\ C_0 &= 47\,200\ \mathrm{ft/h}, \text{ and} \\ H &= 5{\cdot}818 \times 10^{-2}\ \mathrm{ft/h^{\frac{1}{2}}}. \end{aligned}$$

Using these values, we find that r_e/R is extremely small, being only $5{\cdot}8 \times 10^{-4}$ at $R = 5$ cm. We also find that r_e/R is proportional to R, or

$$r_e = 1{\cdot}16 \times 10^{-4} R^2, \tag{26}$$

when both r_e and R are expressed in cm. The insignificant size of the dry-patch under these boiling conditions is not surprising. Owing to the high boiling temperature ($t_{wo} = 1374^\circ$F) and low boiling pressure, the density of the vapor in the bubble is very low; and, since the vapor generated by the microlayer is only about half of the total, the relative amount of liquid evaporated from the microlayer will be extremely small.

A more typical case of liquid-metal boiling is one in which $n = 0{\cdot}9$ for most of the bubble's growth. Such a case [13] is sodium boiling on a smooth Type-316-stainless-steel plate at 1 atm and where $(t_{wo} - t_{sat}) = 100^\circ$F. If we apply equation (23) to this case, a good approximation of dry-patch size is obtained if we integrate the RHS assuming $n = 1$. If we do that, we obtain

$$\frac{H}{S_{0\cdot9}}\left(\frac{r_e}{C_0}\right)^{1/(1\cdot8)} = r_e\left\{\left(\frac{R}{r_e}\right)^{\frac{1}{2}}\left(\frac{R}{r_e} - 1\right)^{\frac{1}{2}} + \ln\left[\left(\frac{R}{r_e}\right)^{\frac{1}{2}} + \left(\frac{R}{r_e} - 1\right)^{\frac{1}{2}}\right]\right\}, \tag{27}$$

which gives the radius of the dry-patch as a function of R. For this particular case [13], we have

$$\begin{aligned} \xi_{0\cdot9} &= 1{\cdot}2489\ \mathrm{ft/(h^{\frac{1}{2}}\,.\,^\circ F)}, \\ S_{0\cdot9} &= 2{\cdot}015 \times 10^{-4}, \\ C_0 &= 16\,900\ \mathrm{ft/h^{0\cdot9}}, \text{ and} \\ H &= 5{\cdot}727 \times 10^{-2}\ \mathrm{ft/h^{\frac{1}{2}}}. \end{aligned}$$

Using these values, equation (27) was used to calculate r_e/R as a function of R, and the results are shown in Fig. 3.

An exact integration of the RHS of equation (23) can be obtained for $\frac{1}{2} < n < 1$ by letting

$$\phi = \cos^{-1}\sqrt{[1 - (r_e/R)^{1/n}]} \tag{28}$$

and transforming the equation to

$$\frac{H}{S_n}\left(\frac{r_e}{C_0}\right)^{1/(2n)} = 2nr_e \int_{\cos^{-1}\sqrt{[1-(r_e/R)^{1/n}]}}^{\pi/2} \frac{\mathrm{d}\phi}{\sin^{2n+1}\phi}. \tag{29}$$

By arbitrarily choosing values of r_e/R, the RHS of this equation can be integrated numerically to give r_e and r_e/R as functions of n and R. For the above case where $n = 0{\cdot}9$, equation (29) is also plotted in Fig. 3. The results fall 6 per cent below those given by (the approximate) equation (27).

In this typical case of sodium boiling on a smooth stainless-steel plate under a pressure of 1 atm, and where $(t_{wo} - t_{sat}) = 100^\circ$F, we see that the relative size of the dry-patch is appreciably greater than that for the earlier case where $n = 1$. However it is still small compared to the area of the microlayer. For example, at $R = 5$ cm, which is much greater than the estimated bubble departure size, r_e/R is only 0·2, making the area of the dry-patch only 4 per cent of the bubble base. Owing to the many assumptions made in the derivation

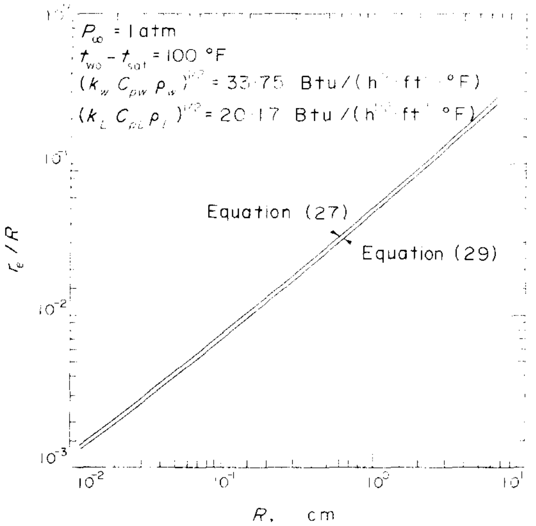

FIG. 3. Curves showing r_e/R as a function of R, for nucleate boiling of sodium on a smooth Type-316-stainless-steel plate. Equations (27) and (29) represent approximate and exact integrations, respectively, of equation (23).

of equation (23), both curves in Fig. 3 should be considered as only approximately correct.

It is interesting to note that the present method of estimating the dry-patch size in hemispherical bubble growth in liquid metals shows that, as n varies from $\frac{1}{2}$ to 1, r_e/R varies from the zeroth to the first power of R. For $n = 0{\cdot}9$, the curves in Fig. 3 indicate that r_e/R is essentially proportional to $R^{0{\cdot}85}$.

Extent of evaporation from the microlayer

If, as we have just shown, the area of the dry-patch is negligible compared with the base area of the bubble, the total volume V_L of liquid evaporated from the microlayer up to any time θ can be expressed by

$$V_L = 2\pi \int_0^R (\delta_0 - \delta) r \, dr, \tag{30}$$

where $0 \leqslant r \leqslant R$, and R is the radius of the bubble at time θ. If we now substitute equation (21) into (30), we get

$$V_L = 2\pi S_n \int_0^R \left[\int_r^R \frac{R^{1/(2n)}}{(R^{1/n} - r^{1/n})^{\frac{1}{2}}} \, dR \right] r \, dr, \tag{31}$$

which, after integrating, takes the simple form

$$V_L = \tfrac{2}{3}\pi R^3 S_n n[B(2n, \tfrac{1}{2})], \tag{32}$$

where $B(2n, \frac{1}{2})$ is the same beta function that appears in equation (12), values of which are given in Table 1.

The total volume of microlayer formed up to time θ is obviously

$$V_{mo} = \int_0^R \delta_0 2\pi r \, dr. \tag{33}$$

Expressing δ_0 in terms of r by means of equations (1) and (3), substituting the result in equation (33), and then integrating, gives

$$V_{mo} = \frac{4\pi n K \sqrt{\nu}}{(4n+1)(C_0)^{1/(2n)}} R^{(4n+1)/2n}. \tag{34}$$

And dividing this into equation (32) gives

$$\frac{V_L}{V_{mo}} = \frac{(4n+1)(C_0)^{1/(2n)} S_n [B(2n, \frac{1}{2})] R^{(2n-1)/2n}}{6K\sqrt{\nu}}, \tag{35}$$

which is the fraction of the microlayer that is evaporated by the time the bubble grows to radius R.

Let us now apply equations (32) and (35) to the specific case of sodium boiling on a Type-316-stainless-steel surface, under a pressure of 100 mm Hg, and with a bubble nucleation radius of 5×10^{-4} in. And let us calculate V_L and V_L/V_{mo} over the period it takes for the bubble to grow to a radius of 1 cm, which is estimated [7] to be well below the final radius of the bubble. For these conditions, it is estimated [13] that

$t_{wo} - t_{sat} = 137°\text{F}$, $n \approx 1$, $C_0 = 48\,800$ ft/h, $K = 0{\cdot}602$ [3], $S_1 = 3{\cdot}208 \times 10^{-5}$, $\nu = 0{\cdot}00894$ ft²/h, and $B[(2n, \frac{1}{2})] = \frac{4}{3}$ (Table 1).

Using these values, $V_L = 8{\cdot}96 \times 10^{-5}$ cm³ and $V_L/V_{mo} = 0{\cdot}025$. In other words, only 2·5 per cent of the original microlayer formation evaporated into the bubble. This is consistent with the conclusion reached in the previous section that the dry-patch area in sodium boiling is negligibly small.

From equation (32), we readily obtain an expression for the volume of vapor produced by evaporation of the microlayer up to any time θ (or value of R). It is

$$V_v = \tfrac{2}{3}\frac{\rho_L}{\rho_v}\pi R^3 S_n n[B(2n, \tfrac{1}{2})]. \tag{36}$$

The fraction of vapor in the bubble (at any time θ) produced by the microlayer is therefore

$$\frac{V_v}{\text{Volume of bubble}} = \frac{\rho_L}{\rho_v} S_n n[B(2n, \tfrac{1}{2})]. \tag{37}$$

Eliminating S_n by means of equation (20) from this equation leads to

$$\frac{V_v}{\text{Volume of bubble}} = \frac{1}{1 + \left[\dfrac{3k_L C_{pL}\rho_L}{k_w C_{pw}\rho_w}\right]^{\frac{1}{2}} \Big/ n[B(2n, \frac{1}{2})]} \tag{38}$$

$$= \frac{1}{1 + Q_{cs}/Q_{ml}}. \tag{39}$$

For the illustrative example given earlier in this section, this ratio is 0·52. Since the area of the curved dome of the bubble is twice that of its base, this says that the average value of q_{cs} was roughly half that of q_{ml} during the time it took the bubble diameter to reach 2 cm. The main reason why q_{cs} is so much lower than q_{ml} is that the time variable in equation (14) is θ, while in equation (8) it is $(\theta - \theta_g)$.

CONCLUDING REMARKS

To the authors' knowledge, the presence of dry-patches in the centers of hemispherical bubbles has never been detected with liquid metals, that is, the characteristic double-rise and double-fall patterns in the heating-surface temperature profiles (see, for example, Cooper and Lloyd [6]) have not been observed. Deane and Rohsenow [8] looked for such profiles, when boiling sodium on a 2·5-in-diam disk that contained a single artificial cylindrical cavity, but found none. If the dry-patch area in boiling sodium is as small as predicted here, it is doubtful that the temperature sensing and recording equipment usually employed is capable of detecting it. Heating surface temperatures are measured by means of small thermocouples imbedded below the heating surface.

On the basis of the simplified calculational model used in the present study, and on the basis of the equations developed therefrom, we conclude that in hemispherical bubble growth in nucleate boiling of liquid metals on smooth metallic surfaces, the area of the dry-patch is negligibly small compared to the base area of the bubble. This is consistent with the very low evaporation rates of the microlayer found in the present study. The three apparent reasons for these liquid-metal results are: (1) n is usually much nearer 1 than $\frac{1}{2}$, (2) the evaporation rate from the liquid at the hemispherical interface is estimated to be roughly the same as that from the microlayer, and (3) the vapor densities are low.

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EVAPORATION DE LA MICROCOUCHE AU COURS DE LA CROISSANCE D'UNE BULLE DANS L'EBULLITION NUCLEEE DES METAUX LIQUIDES

Résumé—L'article fait appel à une analyse thermique afin d'estimer l'évaporation de la microcouche au cours du développement de bulles hémisphériques, pour l'ébullition nucléée de métaux liquides sur des surfaces chauffées. A mesure que la bulle se développe, l'évaporation du film liquide produit en son centre une zone sèche, dont les dimensions dépendent des propriétés physiques et thermiques du système, de la rugosité de la surface chauffante, et de la pression d'ébullition. On a trouvé que l'aire de cette zone relativement à celle du film liquide (ou de la base de la bulle) est habituellement très petite pour les métaux liquides, et peut être négligée dans la plupart des analyses théoriques de formation de bulles. Il a été trouvé de plus que la perte de liquide de la microcouche par évaporation dans la bulle atteint au plus quelques pour cent, dans les cas courants.

Etant donné que le modèle de calcul et l'analyse mathématique supposent un certain nombre d'hypothèses simplificatrices, les résultats numériques de ce travail exploratoire doivent être considérés comme approchés.

VERDAMPFUNG DER MIKROSCHICHT BEI HALBKUGELFÖRMIGEM BLASENWACHSTUM BEIM BLASENSIEDEN VON FLÜSSIGEN METALLEN

Zusammenfassung—Mit Hilfe einer thermischen Analyse wird der Bereich der Verdampfung der Mikroschicht bei halbkugeligem Blasenwachstum beim Blasensieden in Flüssigmetallen an einer beheizten Fläche abgeschätzt. Beim Anwachsen der Blase bedingt die Verdampfung der Mikroschicht eine Trockenstelle in ihrem Mittelpunkt, deren Größe abhängig ist von den thermischen und physikalischen Eigenschaften des Systems, der Rauhigkeit der Heizfläche und dem Siededruck. Es ergab sich, daß die Fläche dieser Trockenstelle im Verhältnis zu der der Mikroschicht (oder Blasenbasis) für Flüssigmetalle sehr klein ist und in den meisten theoretischen Analysen über das Blasenwachstum vernachlässigt werden kann. Es ergab sich ferner, daß der Flüssigkeitsverlust aus der Mikroschicht infolge von Verdampfung in die Blase hinein in typischen Fällen nur einige wenige Prozent beträgt.

Da sowohl das Rechenmodell als auch die mathematische Analyse eine Reihe von vereinfachenden Annahmen enthalten, müssen die numerischen Ergebnisse dieser Anfangsstudie als angenähert betrachtet werden.

ИСПАРЕНИЕ МИКРОСЛОЯ ПРИ РОСТЕ ПОЛУСФЕРИЧЕСКИХ ПУЗЫРЕЙ ВО ВРЕМЯ ПУЗЫРЬКОВОГО КИПЕНИЯ ЖИДКИХ МЕТАЛЛОВ

Аннотация — Применяется тепловой анализ для определения размеров испарения микрослоя при росте полусферических пузырей во время пузырькового кипения жидких металлов на нагретых поверхностях. По мере того как растет пузырь, испарение микрослоя вызывает образование сухого пятна в его центре, размеры которого зависят от тепловых и физических характеристик системы, шероховатости поверхности нагрева и давления кипения. Найдено, что отношение площади этого сухого пятна к площади микрослоя (или основанию пузыря) обычно является очень малым для жидких металлов и им можно пренебречь в большинстве теоретических анализов роста пузыря. Далее было найдено, что потеря жидкости из микрослоя в пузырь за счет испарения составляет в большинстве случаев несколько процентов для типичной ситуации.

Так как и расчетная модель, и математический анализ потребовали ряд упрощающих допущений, численные результаты этого исследования следует считать приближенными.